Roll No. ....

92006

B. Sc. 3rd Semester (Mathematics)

Examination – November, 2014

ADVANCED CALCULUS

Paper : BM-231

Time : Three hours] *[Maximum Marks : 40*

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt *five* questions in all, selecting *one* question from each Unit. All questions carry equal marks. Question No. 9 is ***compulsory***.

UNIT – I

1. (a) A function f defined on R is continuous on R iff for each closed set A in R, $f^{-1}(A)$ is also closed in R.

(b) Prove that the function $f(x) = 2x^2 + 3x - 4$ is uniformly continuous on $[-2, 2]$.

2. (a) State and prove Rolle's Theorem.

(b) Evaluate :

$$\lim_{x \to \infty} x^n e^{-x}, n \in N$$

UNIT – II

3. (a) Let $f : R^2 \to R$ be a function defined as $f(x,y) = \begin{cases} \dfrac{xy}{x^2+y^2} & ; (x,y) \neq (0,0) \\ 0 & ; (x,y) = (0,0) \end{cases}$ show that $\lim_{(x,y) \to (0,0)} f(x,y)$ does not exist.

(b) If $u = f(r)$ where $x = r\cos\theta$, $y = r\sin\theta$ P. T. :

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{1}{r} f'(r)$$

4. (a) State and prove Euler's Theorem on homogeneous functions in x & y of degree n.

(b) Expand $x^4 + x^2y^2 - y^4$ about the Pt (1, 1) upto the terms of 2nd degree.

UNIT – III

5. (a) If $f : R^2 \to R$ be a function such that both f_x and f_y are differentiable at the Pt. (a, b) of the domain then prove that :

$$f_{xy}(a,b) = f_{yx}(a,b)$$

(b) Show that the function :

$$f(x,y)=\begin{cases}\dfrac{x^2y^2}{x^2+y^2} & ; \ (x,y)\neq(0,0)\\ 0 & ; \ (x,y)=(0,0)\end{cases}$$

$f_{xy}(0,0)=f_{yx}(0,0)$ even though the conditions of Schwarz's Theorem and also of Young's Theorem are not satisfied.

6. (a) A rectangular box without top is to have volume 32 cubic feet. Find the dimensions of the box requiring least material for its construction.

(b) Find the minimum value of the function $x^2+y^2+z^2$ subject to condition $ax + by + cz = P$.

UNIT – IV

7. (a) Find the equation of osculating plane of the curve $x = 2 \log t, y = 4t; z = 2t^2 - 1$.

(b) Find the curvature and torsion of the curve given by :

$$\vec{r}=(at-a\sin t, a-a\cos t, bt)$$

8. (a) Show that the radius of spherical curvature of a circular helix $x = a \cos \theta, y = a \sin \theta, z = a\, \theta \cot \alpha$ is equal to the radius of circular curvature.

(b) Find the equation osculating circle at (1, 2, 3) on the curve $x = 2t + 1,\ y = 3t^2 + 2,\ z = 4t^3 + 3$.

Compulsory Question

9. (a) Define the continuity of a function on $[a, b]$.

(b) State the Cauchy Mean Value Theorem.

(c) Evaluate :

$$\lim_{x\to 0} \frac{x - \sin x}{\tan^3 x}$$

(d) If $u = e^{x^y}$, then find :

$$\frac{\partial u}{\partial x} \& \frac{\partial u}{\partial y}$$

(e) Examine for extreme values :

$$f(x,y) = 3x^2 - y^2 + x^3$$

(f) Find the length of circular helix :

$$\vec{r}(t) = a\cos t\,\hat{i} + a\sin t\,\hat{j} + ct\,\hat{k}$$

from $(a, 0, 0)$ to $(a, 0, 2\pi c)$